Constrained diffeomorphometry in computational anatomy

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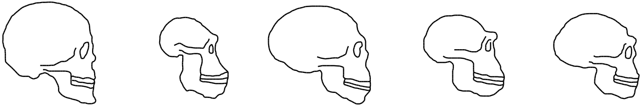
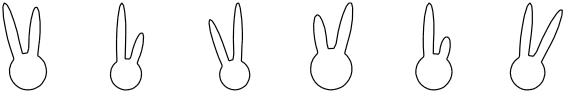
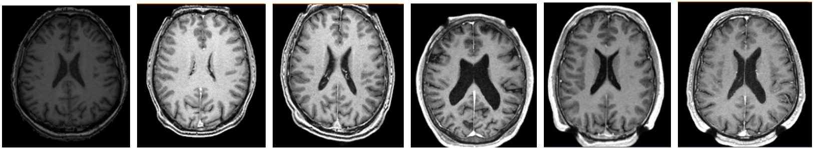
Diffeomorphometry

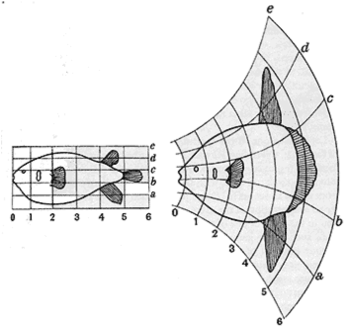
Studying populations of shapes



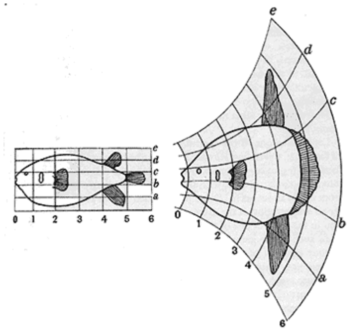




Idea: characterizing the difference between two shapes thanks to the "best" diffeomorphism transforming one into the other.



D'Arcy Thompson (On Growth and Form, 1917)



$$E_{S,T}(\phi) = c(\phi) + \lambda D(\phi \cdot S, T)$$

- Source S, target T
- $D(\phi \cdot S, T)$: data attachment $\longrightarrow$ (i) Local distance
- $c(\phi)$: regularization $\longrightarrow$ (ii) **Deformation model**
- λ : balance factor

Possible shape spaces

- Point clouds
  Euclidean, Hausdorff, measures distances
- Curves, surfaces
  Current, varifolds distances[1][2]
- Images
  L^2 *metrics*
- Combinations

[1] Charon, N., Trouvé, A. (2013). The varifold representation of nonoriented shapes for diffeomorphic registration. SIAM Journal on Imaging Sciences, 6(4), 2547-2580.

[2] Kaltenmark, I., Charlier, B., Charon, N. (2017). A general framework for curve and surface comparison and registration with oriented varifolds. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition.

Theorem

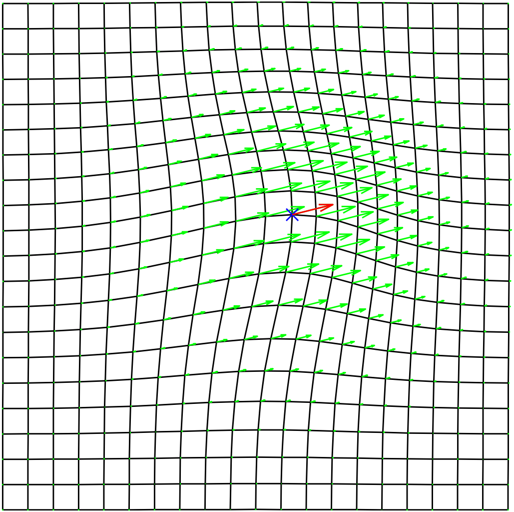
Let $v \in L^1([0,1], C_0^1(\mathbb{R}^d, \mathbb{R}^d))$*, then*

$$\begin{cases} \varphi^v_{t=0} & = & Id \\ \partial_t \varphi^v_t & = & v_t \circ \varphi^v_t \end{cases}$$

has a unique continuous solution called the flow of v*.*
For all t*,* φ^v_t *is a diffeomorphism.*

J. Glaunes (2005). Transport par difféomorphismes de points, de mesures et de courants pour la comparaison de formes et l'anatomie numérique.

$$\begin{cases} \varphi_{t=0} & = & Id \\ \partial_t \varphi_t & = & v_t \circ \varphi_t \end{cases}$$



Shape registration

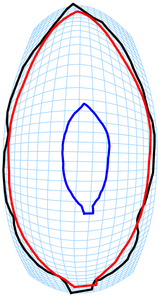
$$\min_{v \in U \subset L^1([0,1], C_0^1(\mathbb{R}^d, \mathbb{R}^d))} \left\{ \int_0^1 c(v_t) \mathrm{d}t + \lambda D(\varphi_{t=1}^v \cdot S, T) \right\}$$

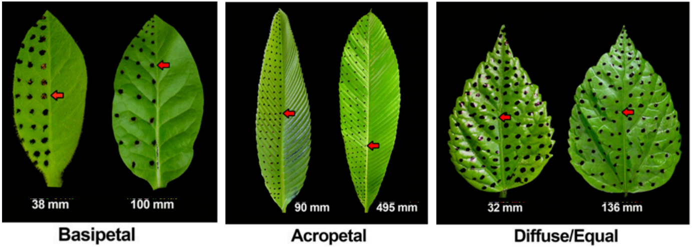
Large deformation diffeomorphic metric mappings (LDDMM)[3] [4]

$$\min_{v \in L^2([0,1], V)} \left\{ \int_0^1 |v_t|_V^2 \mathrm{d}t + \lambda D(\varphi_{t=1}^v \cdot S, T) \right\}$$

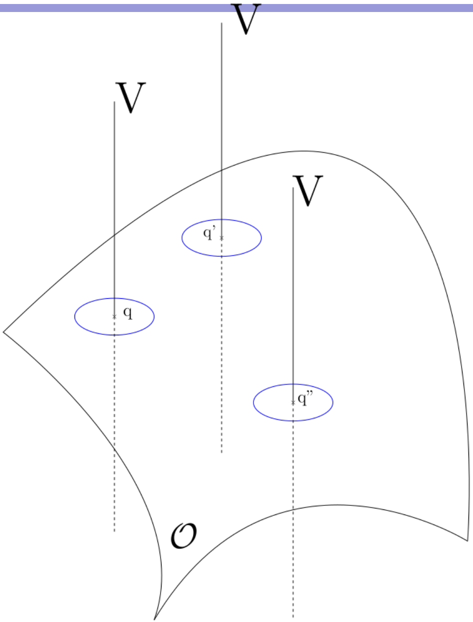
[3] Beg, M. F., Miller, M. I., Trouvé, A., Younes, L. (2005). Computing large deformation metric mappings via geodesic flows of diffeomorphisms. International journal of computer vision

[4] Arguillere, S., Trélat, E., Trouvé, A., Younes, L. (2015). Shape deformation analysis from the optimal control viewpoint. Journal de mathématiques pures et appliquées





[Gupta, M. D., Nath, U. (2015). Divergence in patterns of leaf growth polarity is associated with the expression divergence of miR396. The Plant Cell, tpc-15.]


V
V
V
q'
q
q"
$\mathcal{O}$

Incorporating a structure in deformation models

- Extracting *geometrical descriptors* = relevant data for the structure
- Defining a field structure associated to each geometrical descriptors and an associated cost
- Specifying the evolution of the structure during flow integration

Two main possibilities :

- Defining local field generators[5][6][7][8][9]
- Setting shape-dependent metric on the space of vector fields[10][11]

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[5] S. Durrleman, M. Prastawa, G. Gerig, and S. Joshi. Optimal data-driven sparse parameterization of diffeomorphisms for population analysis. In Information Processing in Medical Imaging , 2011

[6] U. Grenander , A. Srivastava , S. Saini. A pattern-theoric characerization of biological growth. IEEE, 2007

[7] V. Arsigny, X. Pennec, N. Ayache. Polyrigid and Polyaffine Transformations: A Novel Geometrical Tool to Deal with Non-rigid Deformations – Application to the Registration of Histological Slices. Medical Image Analysis. 2005

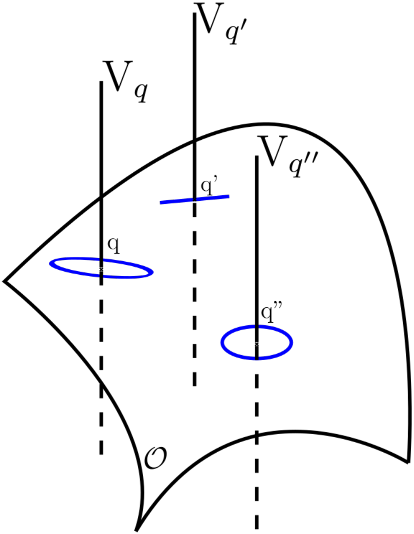
[8] L. Younes. Constrained diffeomorphic shape evolution. Foundations of Computational Mathematics, 2012.

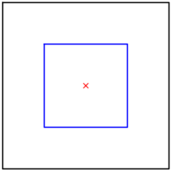
[9] Higher order momentum [S. Sommer M. Nielsen, F. Lauze, and X. Pennec. Higher-order momentum distributions and locally affine lddmm registration. SIAM Journal on Imaging Sciences, 2013]

[10] N. Charon and L. Younes. "Shape spaces: From geometry to biological plausibility. 2022

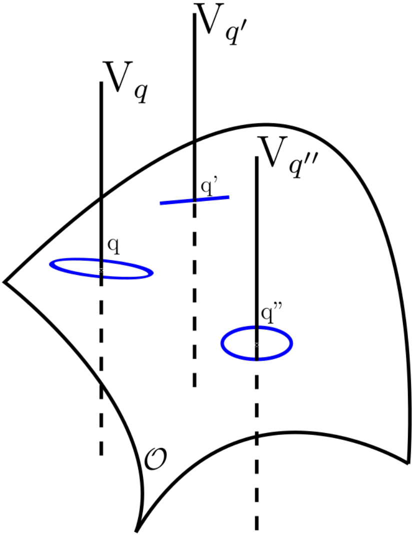
[11] D. N. Hsieh, S. Arguillère, N. Charon, M.I. Miller, L. Youne. A model for elastic evolution on foliated shapes. In International Conference on Information Processing in Medical Imaging. 2019.

DEFORMATION MODULE





- Extend space of shape $q = (\tilde{q}, \theta)$
- $v_q : h \in H \longrightarrow v_{q,h} \in V_q$
- cost : $|v_{q,h}|^2 \leq Mc_q(h)$
- Combination:
  - $q = (\tilde{q}, \theta, \psi)$
  - $V_q = V_\theta + V_\psi$
- Trajectories *s.t.* $\exists v_t \in V_q$: $\dot{q}_t = (v_t \cdot \tilde{q}_t, v_t \cdot \theta_t, v_t \cdot \psi_t, \dots)$

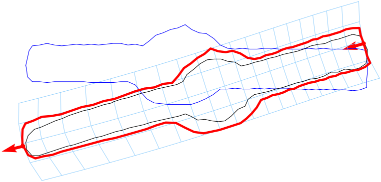


Modular registration

$$J(q,h) = \int c_q(h) + \lambda D(\varphi_{t=1}^{v_{q,h}} \cdot S, T)$$

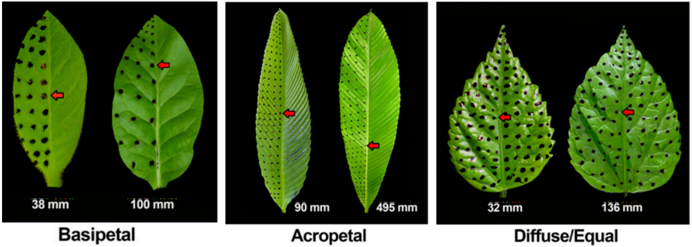
with $\dot{q}_t = v_{q_t,h_t} \cdot q_t$.

- Defining modules
- Minimizing J



Modules:

- Pose:
  - Global translation
  - Global rotation
- Strap lengths:
  - local translations with transported direction



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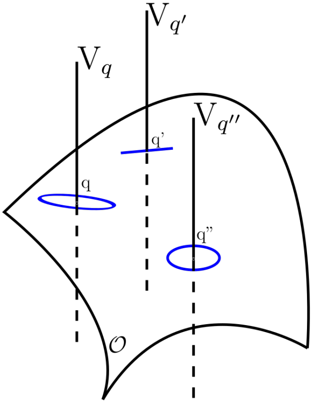


- Defining $v_q : H \mapsto V_q$ from properties to satisfy

$$v_{q,h} = \text{argmin}\{|v|_V^2 + \frac{1}{\nu}|S_q(v) S_q(v) - A_q(h) A$$

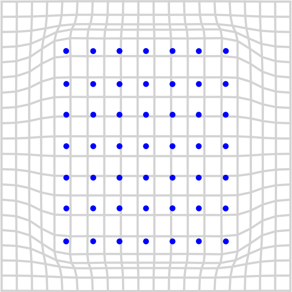
- Evaluation operator S
- Observation operator A
- Model: S and A
- Explicit expression for $v_{q,h}$

$$\zeta_q(h) = argmin\{|v|_V^2 + \frac{1}{\nu}|S_q(v) - A_q(h)|^2\}$$

- $\epsilon_x(v) = \frac{Dv(x)+Dv(x)^T}{2}$

  $\longrightarrow$ Captures local metric changes induced by $Id + v$ around x

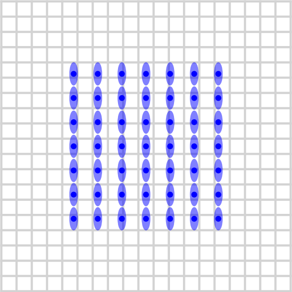
  $\longrightarrow$ Diagonalizable in orthonormal basis
- $q = ((x_1, R_1), \dots, (x_N, R_N)) \in (\mathbb{R}^d \times SO_d(\mathbb{R}))^N$
- $S_q(v) = (\epsilon_{x_i}(v))_i$
- $A_q(h) = \left(R_i \begin{pmatrix} \alpha_i(h) & 0 \\ 0 & \beta_i(h) \end{pmatrix} R_i^{-1}\right)_i$

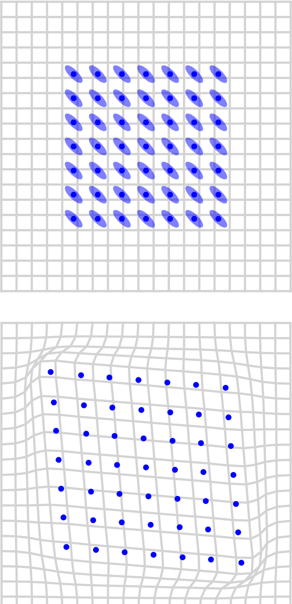


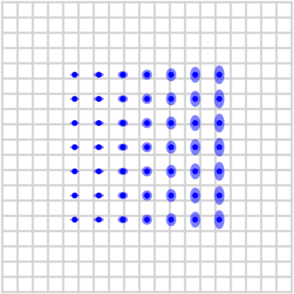
$$A_q(h) = \left(hR_i \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} R_i^{-1} \right)_i$$

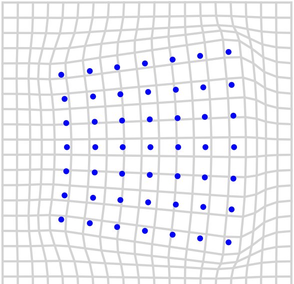
$$R_i = I_2$$

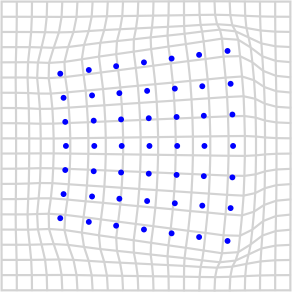
$$\zeta_q(h) = argmin\{|v|_V^2 + \frac{1}{\nu}|S_q(v) - A_q(h)|^2\}$$





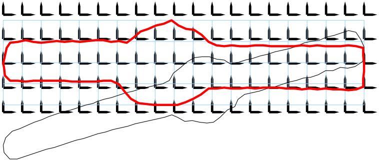






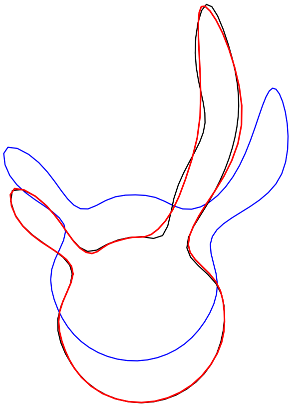
$$A_q(h) = \left(h R_i \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} R_i^{-1} \right)_i$$

$$v \cdot R_i \doteq \frac{Dv - Dv^*}{2} R_i$$



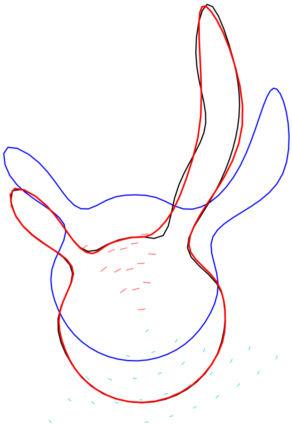
Modules:

- Pose:
  - Global translation
  - Global rotation
- Implicit module:
  - strap lengths : horizontal streching
  - case : isotrope scaling



Modules:

- Pose:
  - Global translation
  - Global rotation
- Implicit module 1:
  - unidirectional stretching
  - parameters: 2 angles, length
- Implicit module 2:
  - Ears position
  - Control of dimension 2



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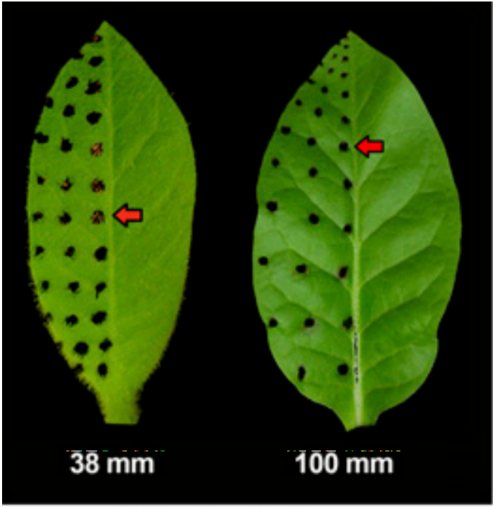
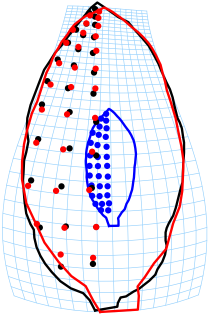
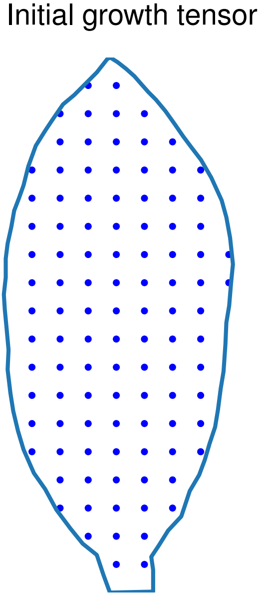
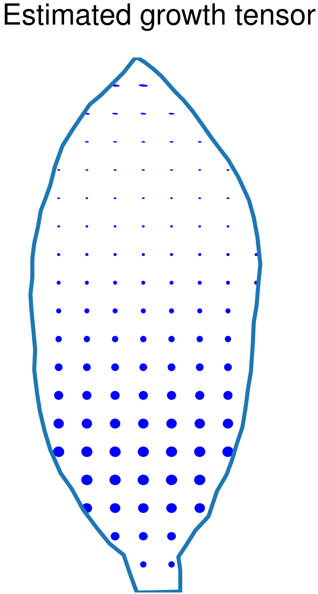


Basipetal



- Joint registration: curves and dots
- Combination of 3 modules :
  - Global translation
  - Implicit of order 1(growth)
  - Unstructured (model correction)


Initial growth tensor


Estimated growth tensor

- Incorporating structures in deformations
  - Implicit deformation modules: simple structure from a biophysical model
  - Estimation and posterior analyze of the structure
- Source and documentation https://github.com/imodal
- *IMODAL: creating learnable user-defined deformation models, Lacroix, Charlier, Trouvé, Gris, CVPR, 2021.*

Thank you for your attention !
Questions ?

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